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Radiative Corrections in a Vector-Tensor Model

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Abstract

In a recently proposed model in which a vector non-Abelian gauge field interacts with an antisymmetric tensor field, it has been shown that the tensor field possesses no physical degrees of freedom. This formal demonstration is tested by computing the one-loop contributions of the tensor field to the self-energy of the vector field. It is shown that despite the large number of Feynman diagrams in which the tensor field contributes, the sum of these diagrams vanishes, confirming that it is not physical. Furthermore, if the tensor field were to couple with a spinor field, it is shown at one-loop order that the spinor self-energy is not renormalizable, and hence this coupling must be excluded. In principle though, this tensor field does couple to the gravitational field.

Recently, a model has been considered in which a non-Abelian gauge field W_μ^a interacts with an antisymmetric tensor field $\phi_{\mu\nu}^a$ with the Lagrange density [1]

$$\begin{aligned} L = -\frac{1}{4}F_{\mu\nu}^a F_{\mu\nu}^a \quad &+ \quad \frac{1}{12}G_{\mu\nu\lambda}^a G_{\mu\nu\lambda}^a + \frac{m}{4}\epsilon_{\mu\nu\lambda\sigma}\phi_{\mu\nu}^a F_{\lambda\sigma}^a \\ &+ \quad \frac{\mu^2}{8}\epsilon_{\mu\nu\lambda\sigma}\phi_{\mu\nu}^a \phi_{\lambda\sigma}^a \ . \end{aligned} \tag{1}$$

In eq. (1), m and μ are mass parameters,

$$F_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g f^{abc} W_\mu^b W_\nu^c \tag{2}$$

$$G_{\mu\nu\lambda}^a = D_\mu^{ab}\phi_{\nu\lambda}^b + D_\nu^{ab}\phi_{\lambda\mu}^b + D_\lambda^{ab}\phi_{\mu\nu}^b \tag{3}$$

and

$$D_\mu^{ab} = \partial_\mu \delta^{ab} + g f^{apb} W_\mu^p \ . \tag{4}$$

This lagrange density is invariant under the infinitesmal gauge transformation

$$\delta W_\mu^a = D_\mu^{ab}\Omega^b \qquad \delta\phi_{\mu\nu}^a = g f^{abc}\phi_{\mu\nu}^b \Omega^c \ . \tag{5}$$

Both by a canonical analysis using the Dirac constraint formalism [2] and by explicit eliminations of non-physical degrees of freedom, it has been shown that in the Abelian limit, the tensor field in eq. (1) does not possess any physical degrees of freedom.

It was surmised that the full non-Abelian model of eq. (1) also does not contain any physical degrees of freedom associated with the tensor field; this conjecture is what we can test by an explicit calculation. Evaluation of the one-loop contributions to the vector self-energy $\langle W_\mu^a W_\nu^b \rangle$ involves Feynman diagrams with both vertices and propagators associated with the tensor field, and if the tensor field is indeed non-physical, its contributions to this Green's function should all cancel. Below we show that this is in fact what happens.

Working in Euclidean space, the contribution to L in eq. (1) that is bilinear in the fields is

$$L^{(2)} = \frac{1}{2}(W_\lambda, \phi_{\alpha\beta}) \begin{pmatrix} \partial^2 I_{\lambda\sigma} & \frac{m}{2} B_{\lambda,\gamma\delta} \\ \frac{m}{2} A_{\alpha\beta,\sigma} & -\frac{1}{2} I_{\alpha\beta,\gamma\delta}\partial^2 + Q_{\alpha\beta,\gamma\delta} + \frac{\mu^2}{4}\epsilon_{\alpha\beta\gamma\delta} \end{pmatrix} \begin{pmatrix} W_\sigma \\ \phi_{\gamma\delta} \end{pmatrix} \tag{6}$$

where

$$\begin{aligned} I_{\alpha\beta} &= \delta_{\alpha\beta}, \quad L_{\alpha\beta} = \partial_\alpha \partial_\beta, \quad I_{\alpha\beta,\gamma\delta} = \frac{1}{2}\left(\delta_{\alpha\gamma}\delta_{\beta\delta} - \delta_{\alpha\delta}\delta_{\beta\gamma}\right) \\ A_{\mu\nu,\lambda} &= \epsilon_{\mu\nu\kappa\lambda}\partial_\kappa = -B_{\lambda,\mu\nu} \\ Q_{\mu\nu,\lambda\sigma} &= \frac{1}{4}\left(\delta_{\mu\lambda}\partial^2_{\nu\sigma} + \delta_{\nu\sigma}\partial^2_{\mu\lambda} - \delta_{\mu\sigma}\partial^2_{\nu\lambda} - \delta_{\nu\lambda}\partial^2_{\mu\sigma}\right) \\ L_{\mu\nu,\lambda\sigma} &= \epsilon_{\mu\nu,\kappa\lambda}\partial_\kappa\partial_\sigma - \epsilon_{\mu\nu\kappa\sigma}\partial_\kappa\partial_\lambda = -R_{\lambda\sigma,\mu\nu}\ . \end{aligned} \tag{7}$$

We have used a gauge fixing Lagrangian

$$L_{gf} = -\frac{1}{2}(\partial \cdot W^a)^2. \tag{8}$$

The inverse of the matrix M appearing in eq. (6) is

$$M^{-1} = \begin{pmatrix} \frac{I_{\sigma\kappa}}{\partial^2} & -\left(\frac{m}{\mu^2\partial^2}\right) D_{\sigma,\pi\tau} \\ \left(\frac{m}{\mu^2\partial^2}\right) C_{\gamma\delta,\kappa} & \left(\frac{4}{\mu^4}\right)\left(1 - \frac{m^2}{\partial^2}\right) Q_{\gamma\delta,\pi\tau} + \left(\frac{1}{\mu^2\partial^2}\right)\left(-L_{\gamma\delta,\pi\tau} + R_{\gamma\delta,\pi\tau}\right) \end{pmatrix} \tag{9}$$

(This corrects a minor mistake in ref. [1].)

Using eq. (9), the free field propagators can be determined. The Feynman rules needed to determine the contribution of the tensor field to $\langle W_\mu^a W_\nu^b \rangle$ are in fig. 1.

In computing the one-loop corrections to $\langle W_\mu^a W_\nu^b \rangle$, it is necessary to include diagrams in which the external leg involves the mixed propagator $\langle W_\mu^a \phi_{\lambda\sigma}^b \rangle$.

The presence of the tensor $\epsilon_{\mu\nu\lambda\sigma}$ in the Lagrange density of eq. (1) makes straightforward application of dimensional regularization difficult. The aspects of dimensional regularization needed are that shifts of variables of integration in Feynman integrals do not generate surface terms, that massless tadpole integrals of the form $\int \frac{d^n k}{(2\pi)^n} \frac{1}{(k^2)^a}$ vanish, and that

$$\epsilon_{\alpha\beta\gamma\delta}\epsilon_{\mu\nu\lambda\sigma} = \left(\delta_{\alpha\mu}\delta_{\beta\nu}\delta_{\gamma\lambda}\delta_{\nu\sigma} + \ldots\right) \tag{10}$$

where in eq. (10) all 24 terms formed by permutting indices (taking into account the anti-symmetry of $\epsilon_{\alpha\beta\gamma\delta}$) are taken into account. We then use the n dimensional relations $\delta_{\mu\mu} = n$ and

$$\int \frac{d^n k}{(2\pi)^n} k_\mu k_\nu f(k^2) = \frac{1}{n}\delta_{\mu\nu} \int \frac{d^n k}{(2\pi)^n} k^2 f(k^2). \tag{11}$$

It turns out though that no integral over loop momentum has to be performed.

The Feynman diagrams associated with the one-loop corrections to $\langle W^a_\mu W^b_\nu \rangle$ involving the tensor field all vanish individually except for the one of fig. (2). These are individually non-zero, but their sum reduces to an integral of the form

$$\Pi_{\mu\nu}(p) = \int \frac{d^n k}{(2\pi)^n} \int_0^1 dx(1-2x) f(x(1-x), k^2, p^2)(p^2\delta_{\mu\nu} - p_\mu p_\nu). \tag{12}$$

On account of the integral over x, this too vanishes; we thus conclude that only the usual Yang-Mills diagrams contribute to $\langle W^a_\mu W^b_\nu \rangle$ at one-loop order. This result is consistent with the conclusion reached in ref. [1] that there are no dynamical degrees of freedom associated with $\phi^a_{\mu\nu}$.

In view of this peculiar feature of the tensor field, it is interesting to examine how it might couple to matter. Let us consider a spinor field ψ and suppose that there are two interaction forms in addition to those of eq. (1),

$$L_1 = g\bar{\psi}\gamma_\mu \tau^a W^a_\mu \psi \tag{13}$$

a

$$L_2 = h\bar{\psi}\sigma_{\mu\nu}\tau^a \phi^a_{\mu\nu}\psi \tag{14}$$

where $\sigma_{\mu\nu} = \frac{1}{2}[\gamma_\mu, \gamma_\nu]$. One can now examine the one-loop corrections to the self-energy of the spinor $\langle \psi\bar{\psi} \rangle$. It turns out the radiative correction proportional to h^2 and gh contains divergences proportional to

$$D_1 = h^2 \left(\frac{m^2}{\mu^4}\right) p^2 \not{p} \tag{15}$$

and

$$D_2 = gh \left(\frac{m}{\mu^2}\right) p^2 \tag{16}$$

respectively. Neither of these divergences are consistent with renormalizability, and so the tensor-spinor interaction of eq. (14) must be excluded, much as we cannot incorporate the magnetic moment interaction $\bar{\psi}\sigma_{\mu\nu}\tau^a F^a_{\mu\nu}\psi$ into the Lagrange density.

One might well ask what the role of the tensor field might be, seeing as its apparent coupling in eq. (1) to the vector field appears to have no physical effect. However, working from the principle that nothing that is forbidden is allowed, we cannot exclude its presence. If $\phi^a_{\mu\nu}$ were to exist though, it would necessarily couple to the gravitational field, contributing to so-called "dark matter".

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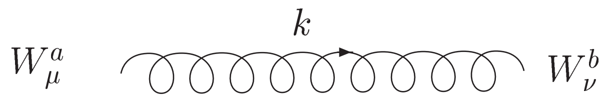
References

[1] S. V. Kuzmin and D. G. C. McKeon, Phys. Lett. B 596 (2004) 301.

[2] P. A. M. Dirac, Can. J. Math 2 (1950) 129, :ibid 3(1951) 1.

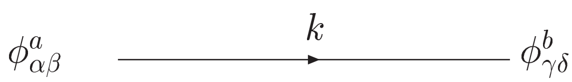
[1]



$$\langle W^a_\mu, W^b_\nu \rangle$$

$$\frac{\delta_{\mu\nu}\delta^{ab}}{k^2}$$

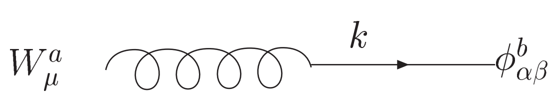
[2]



$$\langle \phi^a_{\alpha\beta}, \phi^b_{\gamma\delta} \rangle$$

$$\frac{\delta^{ab}}{\mu^4}\left(1+\frac{m^2}{k^2}\right)\left(\delta_{\alpha\gamma}k_\beta k_\delta - \delta_{\beta\gamma}k_\alpha k_\delta + \delta_{\beta\delta}k_\alpha k_\gamma - \delta_{\alpha\delta}k_\beta k_\gamma\right)$$
$$+\frac{\delta^{ab}}{\mu^2 k^2}\left(\epsilon_{\alpha\beta\lambda\gamma}k_\delta - \epsilon_{\alpha\beta\lambda\delta}k_\gamma + \epsilon_{\gamma\delta\lambda\alpha}k_\beta - \epsilon_{\gamma\delta\lambda\beta}k_\alpha\right)k_\lambda$$

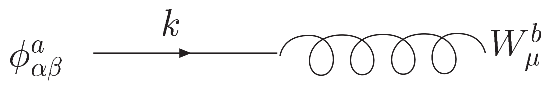
[3]



$$\langle W^a_\mu, \phi^b_{\alpha\beta} \rangle$$

$$-\frac{im\delta^{ab}}{\mu^2 k^2}\left(\delta_{\alpha\mu}k_\beta - \delta_{\beta\mu}k_\alpha\right)$$

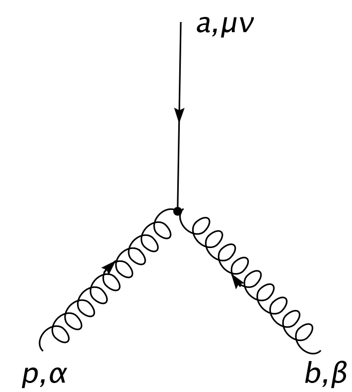
[4]



$$\langle \phi^a_{\alpha\beta}, W^b_\mu \rangle$$

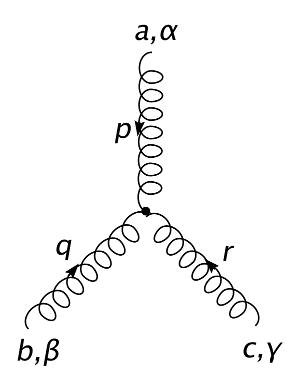
$$\frac{im\delta^{ab}}{\mu^2 k^2}\left(\delta_{\alpha\mu}k_\beta - \delta_{\beta\mu}k_\alpha\right)$$

[5]



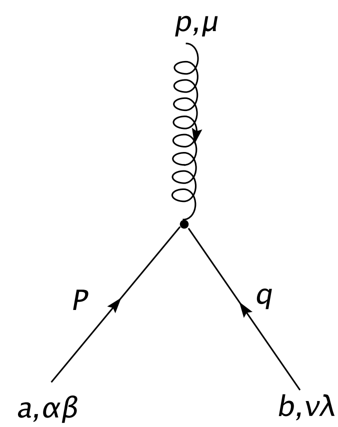
$$\frac{m}{2} g f^{abc} \epsilon_{\mu\nu\alpha\beta}$$

[6]



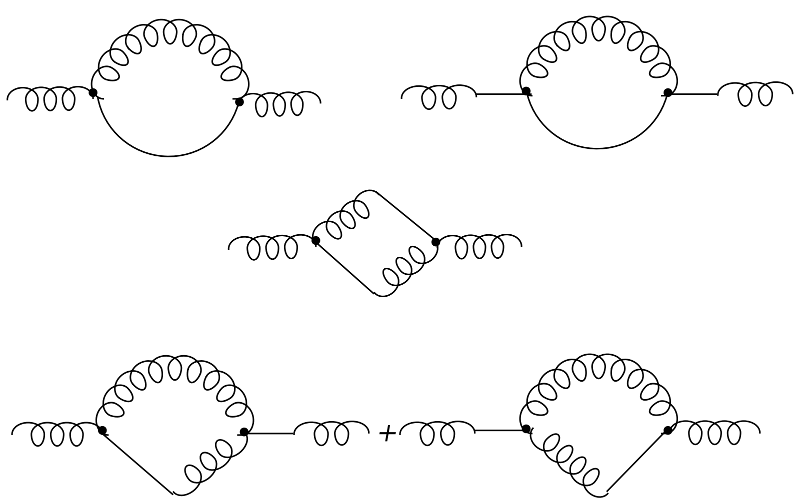
$$igf^{abc}\left[(p-q)_\gamma \delta_{\alpha\beta} + (r-p)_\beta \delta_{\gamma\alpha} + (q-r)_\alpha \delta_{\beta\gamma}\right]$$

[7]



$$\frac{i}{4} g f^{apb} \left[(p-q)_\mu (\delta_{\alpha\nu}\delta_{\beta\lambda} - \delta_{\alpha\lambda}\delta_{\beta\nu}) + p_\nu (\delta_{\alpha\lambda}\delta_{\mu\beta} - \delta_{\beta\lambda}\delta_{\mu\alpha}) \right.$$
$$\left. - p_\lambda (\delta_{\alpha\nu}\delta_{\mu\beta} - \delta_{\beta\nu}\delta_{\mu\alpha}) - q_\alpha (\delta_{\mu\lambda}\delta_{\beta\nu} - \delta_{\mu\nu}\beta_{\beta\lambda}) + q_\beta (\delta_{\mu\lambda}\delta_{\alpha\nu} - \delta_{\mu\nu}\delta_{\alpha\lambda} \right]$$

Feynman Rules
Fig. 1



Non-Vanishing Contributions to $\langle W^a_\mu W^b_\nu \rangle$
Fig. 2